Macromodeling of General Linear Systems Under Stochastic Variations

Yinghao Ye*, Domenico Spina*, Tom Dhaene*, Luc Knockaert* and Giulio Antonini†
*Department of Information Technology (IBCN), Ghent University-iMinds, Gent, Belgium.
Email: {yinghao.ye, domenico.spina, tom.dhaene, luc.knockaert}@intec.ugent.be
†UAq EMC Laboratory, Dipartimento di Ingegneria Industriale e dell'Informazione e di Economia,
Universitá degli Studi dell'Aquila, L'Aquila, Italy.
Email: giulio.antonini@univaq.it

***Abstract*—In this paper, a stochastic modeling approach is proposed for time–domain variability analysis of general linear and passive systems with uncertain parameters. Starting from the polynomial chaos (PC) expansion of the scattering parameters, the Galerkin projections (GP) method is adopted to build an augmented scattering matrix which describes the relationship between the corresponding PC coefficients of the input and output port signals. The Vector Fitting (VF) algorithm is then used to obtain a stable and passive state–space model of such augmented matrix. As a result, a stochastic system is described by an equivalent deterministic macromodel and the time–domain variability analysis can be performed by means of one time–domain simulation. The feasibility, efficiency and accuracy of the proposed technique are verified by comparison with conventional Monte Carlo (MC) approach for a suitable numerical example.**

***Index Terms*—Polynomial chaos, time–domain variability analysis, passivity.**

I. Introduction

The analysis of the effects of geometrical or electrical parameter variability on the performance of integrated circuits (ICs) has drawn great attention in the recent years, due to the increasing integration and miniaturization of such circuits. The MC analysis is the standard technique to investigate this problem, thanks to its accuracy and ease of implementation, but is time and resource intensive, since it requires to perform a large number of simulations.

A different approach is represented by the PC expansion, which has been adopted in many contributions [1] – [4] as efficient alternative to accurate, but computationally cumbersome, MC–based techniques. The PC expansion describes a stochastic process as a suitable summation of orthogonal (polynomial) basis functions with suitable coefficients and gives an analytical representation of the variability of the system with respect to the random variables under consideration [5], [6]. For an extensive reference to PC theory and applications, the reader may consult [1] – [6].

However, it has not been proposed in the literature a technique able to compute a stable and passive PC–based macromodel for time–domain variability analysis of generic linear, passive and frequency dependent microwave systems. Only for specific devices under the effect of stochastic variation (namely multiconductor transmission lines with frequency independent parameters or lumped elements), the technique [2] allows one to compute an equivalent passive deterministic system with respect to the PC coefficients of its port voltages and currents.

Stability and passivity are fundamental model properties to guarantee stable time–domain simulations (e.g. in time–domain circuit simulator such as SPICE–like solvers [7]), which are necessary to analyze the correct behavior of systems in signal integrity and electromagnetic compatibility analysis.

In order to solve such problem, a new stochastic macromodeling approach is proposed, which is capable of computing a time–domain model of the variability of the scattering parameters of generic linear, passive and frequency dependent microwave systems, via the definition of a suitable "augmented system". Such "augmented system" is defined by means of the PC expansion and GP method, while its corresponding macromodel is obtained via the VF algorithm [8].

This paper is structured as follows: Section II describes the proposed technique, which is validated by a suitable numerical example in Section III. Conclusions are shown in Section IV.

II. Stochastic Macromodeling of General Linear Systems

The proposed technique focuses on modeling general linear systems described by scattering parameters and is limited to the case of linear terminations, which can be represented by means of a corresponding impedance or admittance matrix.

Let us consider a system which is under the effect of a vector of random variables $\boldsymbol{\xi}$, then the corresponding scattering parameters $\boldsymbol{S}(\boldsymbol{\xi})$, forward $\boldsymbol{a}(\boldsymbol{\xi})$ and backward $\boldsymbol{b}(\boldsymbol{\xi})$ waves become stochastic quantities, and their relationship can be described as

$$\boldsymbol{b}(\boldsymbol{\xi}) = \boldsymbol{S}(\boldsymbol{\xi})\,\boldsymbol{a}(\boldsymbol{\xi}) \tag{1}$$

$$a^k(\boldsymbol{\xi}) = \frac{V^k(\boldsymbol{\xi}) + Z I^k(\boldsymbol{\xi})}{2\sqrt{|\Re(Z)|}} \tag{2}$$

$$b^k(\boldsymbol{\xi}) = \frac{V^k(\boldsymbol{\xi}) - Z^* I^k(\boldsymbol{\xi})}{2\sqrt{|\Re(Z)|}} \tag{3}$$

where $\boldsymbol{a}(\boldsymbol{\xi}) \in \mathbb{C}^{N_P \times 1} = [a_1(\boldsymbol{\xi}), a_2(\boldsymbol{\xi}), \dots, a_{N_P}(\boldsymbol{\xi})]^T$, $\boldsymbol{b}(\boldsymbol{\xi}) \in \mathbb{C}^{N_P \times 1} = [b_1(\boldsymbol{\xi}), b_2(\boldsymbol{\xi}), \dots, b_{N_P}(\boldsymbol{\xi})]^T$ and $\boldsymbol{S}(\boldsymbol{\xi}) \in \mathbb{C}^{N_P \times N_P}$, $V^k(\boldsymbol{\xi})$ and $I^k(\boldsymbol{\xi})$ are the voltage and current at the k–th port of the system, respectively, while Z is the reference impedance (usually the same for all the system ports). By expressing all the stochastic quantities in (1) $-$ (3) via the PC expansion leads to

$$\sum_{i=0}^{M} \boldsymbol{b}_i \varphi_i(\boldsymbol{\xi}) = \sum_{i=0}^{M}\sum_{j=0}^{M} \boldsymbol{S}_i \boldsymbol{a}_j \varphi_i(\boldsymbol{\xi}) \varphi_j(\boldsymbol{\xi}) \tag{4}$$

$$\sum_{i=0}^{M} a_i^k \varphi_i(\boldsymbol{\xi}) = \frac{\sum_{i=0}^{M} V_i^k \varphi_i(\boldsymbol{\xi}) + \sum_{i=0}^{M} Z I_i^k \varphi_i(\boldsymbol{\xi})}{2\sqrt{|\Re(Z)|}} \tag{5}$$

$$\sum_{i=0}^{M} b_i^k \varphi_i(\boldsymbol{\xi}) = \frac{\sum_{i=0}^{M} V_i^k \varphi_i(\boldsymbol{\xi}) - \sum_{i=0}^{M} Z^* I_i^k \varphi_i(\boldsymbol{\xi})}{2\sqrt{|\Re(Z)|}} \tag{6}$$

where $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, $\boldsymbol{S}_i$, V_i^k and I_i^k are the PC coefficients of the forward and backward wave, of the system scattering matrix, and of the voltage and current at the k–th port of the system, respectively.

Now, by adopting the GP method [2], [6] in equations (4) - (6) allows one to write

$$\boldsymbol{b}_{PC} = \boldsymbol{S}_{PC} \boldsymbol{a}_{PC} \tag{7}$$

where

$$\boldsymbol{a}_{PC} = \frac{1}{2\sqrt{|\Re(Z)|}} (\boldsymbol{V}_{PC} + Z \boldsymbol{I}_{PC}) \tag{8}$$

$$\boldsymbol{b}_{PC} = \frac{1}{2\sqrt{|\Re(Z)|}} (\boldsymbol{V}_{PC} - Z^* \boldsymbol{I}_{PC}) \tag{9}$$

The vectors $\boldsymbol{a}_{PC} \in \mathbb{C}^{(M+1)N_P \times 1}$ and $\boldsymbol{b}_{PC} \in \mathbb{C}^{(M+1)N_P \times 1}$ contain the PC coefficients of the incident and reflected waves, respectively, while $\boldsymbol{V}_{PC} \in \mathbb{C}^{(M+1)N_P \times 1}$ and $\boldsymbol{I}_{PC} \in \mathbb{C}^{(M+1)N_P \times 1}$ are the vectors of the PC coefficients for the port voltages and currents, respectively. Note that the matrix $\boldsymbol{S}_{PC} \in \mathbb{C}^{(M+1)N_P \times (M+1)N_P}$ is a deterministic matrix, obtained via a suitable weighted combination of the PC coefficients of the system scattering matrix and its reciprocity can be preserved by adopting orthonormal basis functions [2]. Furthermore, it is possible to prove that the PC coefficients in (8), (9) are decoupled, as for the original deterministic system: the PC coefficients of the incident and reflected waves at the k–th port of the system depend only on the PC coefficients of voltage and current at the same port.

By comparing (1) $-$ (3) with (7) $-$ (9), a stochastic system depending on the random variables $\boldsymbol{\xi}$ is described via an equivalent deterministic system which still represent a system defined by a scattering parameter matrix, namely $\boldsymbol{S}_{PC}$, with respect to the PC coefficients of the incident and reflected waves $\boldsymbol{a}_{PC}$ and $\boldsymbol{b}_{PC}$. Hence, the condition for the passivity of $\boldsymbol{S}_{PC}$ are identical to the ones for a deterministic scattering parameter matrix [9].

Finally, a time–domain model in a state–space form of the augmented system can be computed via the VF algorithm [8] and its stability and passivity can be enforced by means of standard techniques, such as [10].

Now, the equations of the terminations at the system ports are in the form

$$\boldsymbol{V}(\boldsymbol{\xi}) = \boldsymbol{V}^S - \boldsymbol{Z}^L \boldsymbol{I}(\boldsymbol{\xi}) \tag{10}$$

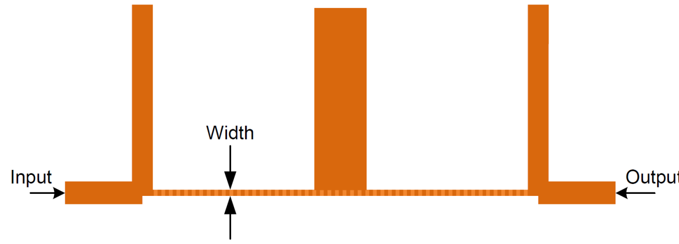

Fig. 1. Geometry of bandstop microstrip filter.

where $\boldsymbol{V}(\boldsymbol{\xi})$, $\boldsymbol{I}(\boldsymbol{\xi})$ represent the vector of the port voltages and currents, respectively, $\boldsymbol{V}^S$ is the matrix of the voltage sources and $\boldsymbol{Z}^L$ is the matrix of the port terminations. Again, by means of the PC expansion and GP method, the following equation can be easily derived

$$\boldsymbol{V}_{PC} = \boldsymbol{V}_{PC}^S - \boldsymbol{Z}_{PC}^L \boldsymbol{I}_{PC} \tag{11}$$

where $\boldsymbol{V}_{PC}$ and $\boldsymbol{I}_{PC}$ are the matrices of the PC coefficients of the port voltages and currents, while the matrix $\boldsymbol{Z}_{PC}^L$ is block diagonal and depends only on the original port terminations $\boldsymbol{Z}^L$. Finally, the PC coefficients of the port signals can be directly determined by combining (7) $-$ (9) with (11) and the time–domain variability analysis can be performed with accuracy and efficiency thanks to the properties of the PC expansion (i.e. stochastic moments of the ports voltages and current can be analytically determined).

III. Numerical Example

The simulations shown in this Section are performed with MATLAB 2015a[1] and ADS 2015[2] on a computer with Intel Core i3 processor and 8GB RAM.

In order to validate the proposed stochastic modeling approach, the bandstop microstrip filter [11] in Fig.1 is studied within the frequency range [100Hz $-$ 7GHz]. The width w of the base microstrip highlighted in Fig.1 and relative permittivity ε_r of the substrate are considered as Gaussian distributed random variables, $w \sim \mathcal{N}(0.5\text{mm}, (60\mu\text{m})^2)$, $\varepsilon_r \sim \mathcal{N}(6.15, (0.18)^2)$, while all the other parameters are chosen as shown in [11].

An initial set of scattering parameters values has been evaluated via ADS, over 25 samples for the geometrical parameters (w, ε_r) chosen over a 5×5 regular grid. Then, the PC coefficients of the filter scattering parameters are computed with $M + 1 = 10$ basis functions and a maximum degree $P = 3$ of the PC basis functions for 101 frequency samples, following the approach described in [4]. Next, $\boldsymbol{S}_{PC}$ can be calculated for each frequency sample as described in Section II. Finally, a stable and passive state–space model of $\boldsymbol{S}_{PC}$ has been obtained by means of the VF algorithm targeting -50dB as maximum absolute model error between the scattering parameters of augmented matrix and the corresponding state–space model.

[1]The Mathworks Inc., Natick, MA, USA.

[2]Advanced Design System (ADS), Keysight Technologies, Santa Rosa, CA.

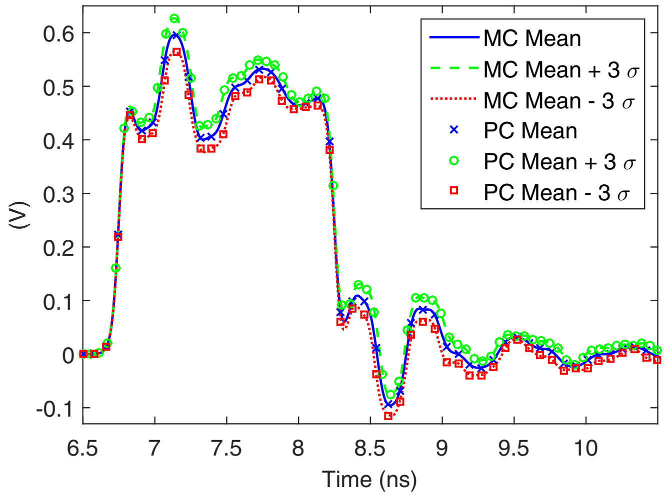

Fig. 2. The mean μ and the range $\mu \pm 3\sigma$ of the input voltage of the bandstop microstrip filter. The (solid, dash, dot) lines are computed using the MC method for 10000 (w, ε_r) samples. The symbols (×,○,□) represent the same quantities obtained with the PC–based augmented system.

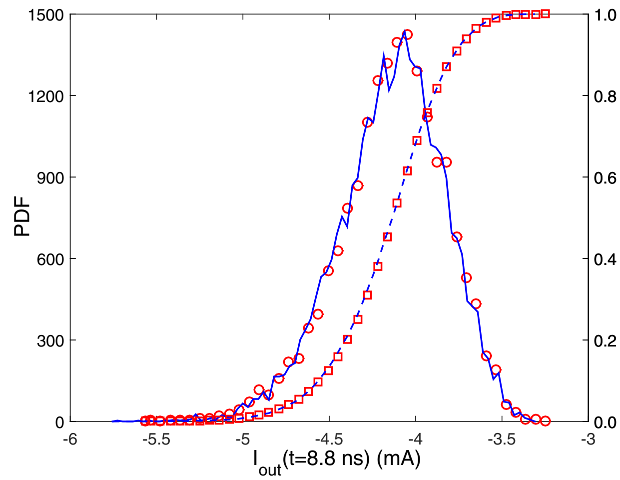

Fig. 3. PDF (full line) and CDF (dashed line) of the output current for $t = 8.8$ns computed using the novel PC–based technique; the circles (○) and squares (□) represent the same quantities computed by the MC method.

The time–domain simulations are performed by exciting the filter with a smooth voltage pulse with amplitude 1V, rise/fall times 0.1ns, width 1.5ns, initial delay 6.75ns and internal resistance $R_S = 50\Omega$, while the filter is terminated on a $R_L = 50\Omega$ resistor. Note that the computation of the PC coefficients of the port signals via the obtained augmented system requires only one time–domain simulation.

In order to validate the efficiency and accuracy of our novel technique, a comparison with the time–domain MC analysis is performed. First, the filter scattering parameters have been computed for 10000 samples of the chosen random variables (w, ε_r) in ADS. Next, a corresponding state–space description is calculated via the VF algorithm for each one of the 10000 scattering matrices obtained so far, by targeting a maximum modeling error of -50dB, and the time–domain MC analysis is performed by means of such state–space models. The proposed technique has a good accuracy compared with the classical MC analysis not only in computing simple stochastic moments, as shown in Fig. 2, but also complex stochastic quantities like the probability density (PDF) and cumulative distribution (CDF) functions, see Fig. 3. Furthermore, the proposed method requires only 21min 39.52s to estimate the filter variability features, while the MC analysis requires 139h 36min 55.94s, corresponding to a simulation speed–up of $386\times$.

IV. Conclusion

An stochastic macromodeling approach for time–domain variability analysis of general linear and passive multiport systems is presented in this paper, which overcomes the limitations of the technique [2]. Based on the PC expansion and GP method, the proposed technique is non–intrusive and can be applied to many different microwave structures (i.e. distributed filters, connectors), since its based on the scattering parameters of the system under study. Note that the time–domain variability analysis can be performed by means of one time–domain simulation of the computed stable and passive macromodel. The accuracy and efficiency of the proposed method are validated by a suitable numerical example, achieving a simulation speedup of $386\times$ with respect to the MC analysis.

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